Review 24 Infinite series

sequence $a_1, a_2, a_3, \ldots$ {individual terms}

series $a_1 + a_2 + a_3 \ldots$

$$\sum_{n=1}^{\infty} a_n$$ {sum of the sequence}

geometric series {common ratio r}

$$a + ar + ar^2 + \cdots + ar^{n-1} + \cdots = \frac{a}{1-r}$$

{if $|r| < 1$}

Apr 7-8:13 AM

p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge if $p > 1$, diverge if $p \leq 1$

$p=2$: $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \cdots$ converge

$p=1$: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \ldots$ harmonic diverges

$\sum \frac{(-k)^n}{n^p}$; $\sum_{n=1} \frac{(-1)^n}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots$

converges by AST

$\sum \frac{-1^n}{n}$ diverges

neg of harmonic

$-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} \cdots$

$-1\left(1 + \frac{1}{2} + \frac{1}{3} \cdots\right)$

Apr 7-8:18 AM

n^{th} term test for divergence

If sequence does not converge to 0 then the series diverges

use with

ratio test: $n!$ or a^n

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$$

if $L<1$ conv.

if $L>1$ div.

if $L=1$?

Apr 7-8:22 AM

Integral Test

$$\sum_{n=1}^{\infty} a_n \quad , \quad \int_a^{\infty} a_n \, dn$$

both behave the same

comparison test

$$0 \le \sum a_n < \sum b_n$$

$\sum b_n$ ← converges

$\sum a_n$ ↑ must also converge

or

$$\sum a_n > \sum b_n \ge 0$$

$\sum a_n$ ↑ must also diverge

$\sum b_n$ ↑ diverges

Apr 7-8:29 AM

absolute convergence

$\sum |a_n|$ conv. so $\sum a_n$ conv

conditional convergence

$\sum |a_n|$ div & $\sum a_n$ conv

use AST

Apr 7-8:32 AM

$$\sum_{n=1}^{\infty} 4\left(\frac{-1}{3}\right)^{n-1} = 4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} \ldots \left| 4\left(\frac{-1}{3}\right)^{n-1} \right|$$

next

AST remainder

$\epsilon < 4\left(-\frac{1}{3}\right)^{n} \leq .001$

solve for n by trial & error

Apr 7-8:35 AM